1 Problems on Group Theory (by Dr. J. G. Mehta)

1. **NET JUNE 2019 (B):** Consider the set of matrices

$$G = \left\{ \begin{pmatrix} s & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{Z}, s \in \{-1, 1\} \right\}.$$

   Then which of the following is true?

   (a) G forms a group under addition

   (b) G forms an abelian group under multiplication.

   (c) Every element in G is diagonalisable over $\mathbb{C}$

   (d) G is finitely generated abelian group under multiplication

2. **NET JUNE 2019 (C):** For any prime p consider group $G = GL_2(\mathbb{Z}/p\mathbb{Z})$. Then which of the following are true?

   (a) G has an element of order p

   (b) G has exactly one element of order p

   (c) G has no p-Sylow subgroup

   (d) Every element of order p is conjugate to a matrix $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where $a \in (\mathbb{Z}/p\mathbb{Z})^*$

3. **NET JUNE 2019 (C):** Let $\mathbb{Z}[x]$ be the ring of polynomials over integers. Then the additive group $\mathbb{Z}[x]$ is

   (a) isomorphic to the multiplicative group $\mathbb{Q}^+$ of positive rational numbers.

   (b) isomorphic to the group of rational numbers $\mathbb{Q}$ under addition.

   (c) countable

   (d) uncountable

4. **NET DEC 2018 (B):** The number of group homomorphisms from the alternating group A_5 to the symmetric group S_4 is

   (a) 1 (b) 12 (c) 20 (d) 6

5. **NET DEC 2018 (B):** Let $p \geq 23$ be a prime number such that the decimal expansion (base 10) of $\frac{1}{p}$ is periodic with period $p-1$ (that is $\frac{1}{p} = 0.\overline{a_1 a_2 \cdots a_{p-1}}$ with $a_i \in \{0, 1, \ldots, 9\}$ for all i) and for any m, $1 \leq m < p-1$, $\frac{1}{p} \neq 0.\overline{a_1 a_2 \cdots a_m}$.

   Let $(\mathbb{Z}/p\mathbb{Z})^*$ denote the multiplicative group of integers modulo p. Then which of the following is correct?

   (a) The order of $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is a proper divisor of $(p-1)$

   (b) The order of $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is $\frac{(p-1)}{2}$

   (c) The element $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is a generator of the group $(\mathbb{Z}/p\mathbb{Z})^*$

   (d) The group $(\mathbb{Z}/p\mathbb{Z})^*$ is cyclic but not generated by the element 10

6. **NET DEC 2018 (C):** For any group G let $\mathrm{Aut}(G)$ denote the group of automorphisms of G. Which of the following are true?

(a) if G is finite, then $\text{Aut}(G)$ is finite.

(b) if G is cyclic, then $\text{Aut}(G)$ is cyclic.

(c) if G is infinite, then $\text{Aut}(G)$ is infinite.

(d) if $\text{Aut}(G)$ is isomorphic to $\text{Aut}(H)$, where G and H are two groups, then G is isomorphic to H.

7. **NET JUNE 2018 (B):** Let S_7 denote the group of permutations of the set $\{1, 2, 3, 4, 5, 6, 7\}$. Which of the following is true?

   (a) There are no elements of order 6 in S_7

   (b) There are no elements of order 7 in S_7

   (c) There are no elements of order 8 in S_7

   (d) There are no elements of order 10 in S_7

8. **NET JUNE 2018 (B):** The number of group homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is

   (a) zero (b) one (c) five (d) ten

9. **NET JUNE 2018 (C):** Let $G = S_3$ be the permutation group of 3 symbols. Then

   (a) G is isomorphic to a subgroup of a cyclic group.

   (b) there exists a cyclic group H such that G maps homomorphically onto H.

   (c) G is a product of cyclic groups.

   (d) there exists a non-trivial homomorphism from G to the additive group $(\mathbb{Q}, +)$ of rational numbers.

10. **NET JUNE 2018 (C):** Let G be a group with $|G| = 96$. Suppose H and K are subgroups of G with $|H| = 12$ and $|K| = 16$. Then

    (a) $H \cap K = \{e\}$

    (b) $H \cap K \neq \{e\}$

    (c) $H \cap K$ is Abelian

    (d) $H \cap K$ is not Abelian

11. **NET DEC 2017 (B):** The group S_3 of permutations of $\{1, 2, 3\}$ acts on the three dimensional vectors space over the finite field $\mathbb{F}_3$ of three elements, by permuting vectors in basis $\{e_1, e_2, e_3\}$ by $\sigma(e_i) = e_{\sigma(i)}$, for all $\sigma \in S_3$. The cardinality of the vectors fixed under the above action is

    (a) 0 (b) 3 (c) 9 (d) 27

12. **NET DEC 2017 (C):** Let G be a finite abelian group and $a, b \in G$ with $\text{order}(a) = m$, $\text{order}(b) = n$. Which of the following are necessarily true?

    (a) $\text{order}(ab) = mn$

    (b) $\text{order}(ab) = \text{lcm}(m, n)$

    (c) there is an element of G whose order is $\text{lcm}(m, n)$

    (d) $\text{order}(ab) = \gcd(a, b)$

13. **NET JUNE 2017 (C):** For an integer $n \geq 2$, let S_n be the permutation group on n letters and A_n the alternating group. Let $\mathbb{C}^*$ be the group of nonzero complex numbers under multiplication. Which of the following are correct statements?

(a) For every integer $n \geq 2$, there is a non trivial homomorphism $\chi : S_n \to \mathbb{C}^*$.

(b) For every integer $n \geq 2$, there is a unique non trivial homomorphism $\chi : S_n \to \mathbb{C}^*$.

(c) For every integer $n \geq 3$, there is a non trivial homomorphism $\chi : A_n \to \mathbb{C}^*$.

(d) For every integer $n \geq 5$, there is no non trivial homomorphism $\chi : A_n \to \mathbb{C}^*$.

14. **NET JUNE 2017 (C):** Let G be a group of order 125. Which of the following statements are necessarily true?

(a) G has a non-trivial abelian subgroup.

(b) The center of G is a proper subgroup.

(c) The center of G has order 5.

(d) There is a subgroup of order 25.

15. **NET DEC 2016 (B):** Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true?

(a) There exists a finite group which is not a subgroup of S_n for any $n \geq 1$.

(b) Every finite group is a subgroup of A_n for some $n \geq 1$.

(c) Every finite group is a quotient of A_n for some $n \geq 1$.

(d) No finite abelian group is a quotient of S_n for some $n \geq 3$.

16. **NET DEC 2016 (B):** What is the number of non-singular 3×3 matrices over $\mathbb{F}_2$?

(a) 168. (b) 384. (c) 2^3. (d) 3^2.

17. **NET DEC 2016 (C):** Consider the following subsets of the group of 2×2 non-singular matrices over $\mathbb{R}$:

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R},\ ad = 1 \right\}$$

$$H = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}.$$

Which of the following statements are correct?

(a) G forms a group under matrix multiplication.

(b) H is a normal subgroup of G.

(c) The quotient group G/H is well-defined and is Abelian.

(d) The quotient group G/H is well defined and is isomorphic to the group of diagonal matrices (over $\mathbb{R}$) with determinant 1.

18. **NET DEC 2016 (C):** Let $\mathbb{C}$ be the field of complex numbers and $\mathbb{C}^*$ be the group of nonzero complex numbers under multiplication. Then which of the following are true?

(a) $\mathbb{C}^*$ is cyclic.

(b) Every finite subgroup of $\mathbb{C}^*$ is cyclic.

(c) $\mathbb{C}^*$ has finitely many subgroups.

(d) Every proper subgroup of $\mathbb{C}^*$ is cyclic.

19. **NET JUNE 2016 (B):** Let $G = (\mathbb{Z}/25\,\mathbb{Z})^*$ be the group units (i.e. the elements that have multiplicative inverse) in the ring $(\mathbb{Z}/25\,\mathbb{Z})$. Which of the following is a generator of G?

    (a) 3. (b) 4 (c) 5 (d) 6

20. **NET JUNE 2016 (C):** Let G be a finite abelian group of order n. Pick each correct statement from below.

    (a) If d divides n, there exists a subgroup of G of order d.

    (b) If d divides n, there exists an element of order d in G.

    (c) If every proper subgroup of G is cyclic, then G is cyclic.

    (d) If H is a subgroup of G, there exists a subgroup N of G such that $G/N \cong H$.

21. **NET JUNE 2016 (C):** Consider the symmetric group S_{20} and its subgroup A_{20} consisting of all even permutations. Let H be a 7-Sylow of A_{20}. Pick each correct statement from below:

    (a) $|H| = 49$.

    (b) H must be cyclic.

    (c) H is a normal subgroup of A_{20}.

    (d) Any 7-Sylow subgroup of S_{20} is a subset of A_{20}.

22. **NET JUNE 2016 (C):** Let p be a prime. Pick each correct statement from below. Up to isomorphism

    (a) there are exactly two abelian groups of order p^2.

    (b) there are exactly two groups of order p^2.

    (c) there are exactly two commutative rings of order p^2.

    (d) there is exactly one integral domain of order p^2.

23. **NET DEC 2015 (B):** What is the total number of positive integer solutions to the equation $(x_1 + x_2 + x_3)(y_1 + y_2 + y_3 + y_4) = 15$.

    (a) 1 (b) 2. (c) 3 (d) 4

24. **NET DEC 2015 (B):** What is the cardinality of the set $\{z \in \mathbb{C} \mid z^{98} = 1 \text{ and } z^n \neq 1 \text{ for any } 0 < n < 98\}$?

    (a) 0 (b) 12. (c) 42 (d) 49

25. **NET DEC 2015 (B):** A group generated by G of elements x, y with the relations $x^3 = y^2 = (xy)^2 = 1$. The order of G is

    (a) 4 (b) 6 (c) 8 (d) 12.

26. **NET DEC 2015 (C):** Let a_n denote the number of those permutations σ on $\{1, 2, \ldots, n\}$ such that σ is a product of exactly two disjoint cycles. Then:

    (a) $a_5 = 50$ (b) $a_4 = 14$ (c) $a_5 = 40$ (d) $a_4 = 11$

27. **NET DEC 2015 (C):** Let G be a simple group of order 60. Then

(a) G has six Sylow-5 subgroups.

(b) G has four Sylow-3 subgroups.

(c) G has cyclic subgroup of order 6.

(d) G has a unique element of order 2.

28. **NET DEC 2015 (C):** For $n \geq 1$, let $(\mathbb{Z}/n\mathbb{Z})^*$ be the group of units of $(\mathbb{Z}/n\mathbb{Z})$. Which of the following groups are cyclic?

(a) $(\mathbb{Z}/10\mathbb{Z})^*$ (b) $(\mathbb{Z}/2^3\mathbb{Z})^*$ (c) $(\mathbb{Z}/100\mathbb{Z})^*$ (d) $(\mathbb{Z}/163\mathbb{Z})^*$

29. **NET JUNE 2015 (B):** How many elements does the set $\{z \in \mathbb{C} \mid z^{60} = -1,\ z^k \neq -1 \text{ for } 0 < k < 60\}$ have?

(a) 24. (b) 30. (c) 32. (d) 45.

30. **NET JUNE 2015 (B):** Upto isomorphism, the number of abelian groups of order 108 is

(a) 12. (b) 9. (c) 6. (d) 5.

31. **NET JUNE 2015 (C):** Let $\sigma : \{1,2,3,4,5\} \to \{1,2,3,4,5\}$ be a permutation (one-one and onto function) such that $\sigma^{-1}(j) \leq \sigma(j)$, $\forall\ j,\ 1 \leq j \leq 5$. Then which of the following are true?

(a) $\sigma \circ \sigma(j) = j$ for all $j,\ 1 \leq j \leq 5$.

(b) $\sigma^{-1}(j) = \sigma(j) = j$ for all $j,\ 1 \leq j \leq 5$.

(c) The set $\{k : \sigma(k) \neq k\}$ has even number of elements.

(d) The set $\{k : \sigma(k) = k\}$ has odd number of elements.

32. **NET JUNE 2015 (C):** If x, y and z are elements of a group such that $xyz = 1$, then

(a) $yzx = 1$. (b) $yxz = 1$. (c) $zxy = 1$. (d) $zyx = 1$.

33. **NET JUNE 2015 (C):** Which of the following cannot be the class equation of a group of order 10?

(a) $1 + 1 + 1 + 2 + 5 = 10$. (c) $1 + 2 + 2 + 5 = 10$.

(b) $1 + 2 + 3 + 4 = 10$. (d) $1 + 1 + 2 + 2 + 2 + 2 = 10$.

34. **NET DEC 2014 (B):** In the group of all invertible 4×4 matrices with entries in the field of 3 elements, any 3-Sylow subgroup has cardinality:

(a) 3. (b) 81. (c) 243. (d) 729.

35. **NET DEC 2014 (B):** The number of conjugacy classes in the permutation group S_6 is

(a) 12 (b) 11. (c) 10 (d) 6

36. **NET DEC 2014 (C):** Let G be a non-abelian group. Then, its order can be:

(a) 25 (b) 55. (c) 125. (d) 35.

37. **NET DEC 2014 (C):** Let G be a group of order 45. Then

(a) G has an element of order 9.

(b) G has a subgroup of order 9.

(c) G has a normal subgroup of order 9.

(d) G has a normal subgroup of order 5.

38. **NET JUNE 2014 (C):** Consider the multiplicative group G of all the (complex) 2^n-th roots of unity, where $n = 0, 1, 2, \ldots$. Then

(a) Every subgroup of G is finite

(b) G has a finite set of generators

(c) G is cyclic

(d) Every finite subgroup of G is cyclic

39. **NET JUNE 2014 (B):** The total number of non-isomorphic groups of order 122 is

(a) 2. (b) 1. (c) 61. (d) 4.

40. **NET DEC 2013 (C):** Determine which of the following cannot be the class equation of a group

(a) $10 = 1 + 1 + 1 + 2 + 5$.

(b) $4 = 1 + 1 + 2$.

(c) $8 = 1 + 1 + 3 + 3$.

(d) $6 = 1 + 2 + 3$.

41. **NET DEC 2013 (B):** How many normal subgroups does a non-abelian group G of order 21 have other than the identity subgroup $\{e\}$ and G?

(a) 0. (b) 1. (c) 3. (d) 7.

42. **NET DEC 2013 (B):** (**Same as NET DEC 2011 (B)**) The number of group homomorphisms from the symmetric group S_3 to the additive group $\mathbb{Z}/6\,\mathbb{Z}$ is

. (a) 1 (b) 2. (c) 3. (d) 0.

43. **NET JUNE 2013 (B):** Let G be a simple group of order 168. What is the number of subgroups of G of order 7?

(a) 1. (b) 7. (c) 8. (d) 28.

44. **NET JUNE 2013 (C):** Let $\sigma = (1\ 2)(3\ 4\ 5)$ and $\tau = (1\ 2\ 3\ 4\ 5\ 6)$ be permutations in S_6, the group of permutations on six symbols. Which of the following statements are true?

(a) The subgroups $\langle\sigma\rangle$ and $\langle\tau\rangle$ are isomorphic to each other.

(b) σ and τ are conjugate in S_6.

(c) $\langle\sigma\rangle \cap \langle\tau\rangle$ is the trivial group

(d) σ and τ commute

45. **NET JUNE 2013 (C):** Let S_n denote the symmetric group on n symbols. The group $S_3 \oplus (\mathbb{Z}/2\mathbb{Z})$ is isomorphic to which of the following groups?

(a) $\mathbb{Z}/12\,\mathbb{Z}$

(b) $(\mathbb{Z}/6\,\mathbb{Z}) \oplus (\mathbb{Z}/2\,\mathbb{Z})$

(c) A_4, the alternating group of order 12

(d) D_6, dihedral group of order 12

46. **NET JUNE 2013 (C):** Consider the two sets $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$. Choose the correct statements.

(a) The total number of functions from A to B is 125

(b) The total number of functions from A to B is 243

(c) The total number of one-one functions from A to B is 60

(d) The total number of one-one functions from A to B is 120

47. **NET DEC 2012 (C):** For a positive integer $n \geq 4$ and a prime number $p \leq n$, let $U_{p,n}$ denote the union of all p-Sylow subgroups of the alternating group A_n on n letters. Also let $K_{p,n}$ denote the subgroup of A_n generated by $U_{p,n}$, and let $|K_{p,n}|$ denote the order of $K_{p,n}$. Then

(a) $|K_{2,4}| = 12$ (b) $|K_{2,4}| = 4$ (c) $|K_{2,5}| = 60$ (d) $|K_{3,5}| = 30$

48. **NET JUNE 2012 (B):** Let $\mathbb{F}$ be a field of 8 elements and $A = \{x \in \mathbb{F} \mid x^7 = 1 \text{ and } x^k \neq 1 \text{ for all natural numbers } k < 7\}$. Then the number of elements in A is

(a) 1. (b) 2. (c) 3. (d) 6.

49. **NET JUNE 2012 (B):** Consider the group $\mathbb{Q}/\mathbb{Z}$, where $\mathbb{Q}$ and $\mathbb{Z}$ are the groups of rationals and integers respectively. Let n be a positive integer. Then there is a cyclic subgroup of order n?

(a) not necessarily.

(b) yes, a unique one.

(c) yes, but not necessarily a unique one.

(d) never

50. **NET JUNE 2012 (B):** The no. of nontrivial ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{28}$ is

(a) 1. (b) 3. (c) 4. (d) 7.

51. **NET JUNE 2012 (C):** For any group G of order 36 and any subgroup H of G of order 4

(a) $H \subset Z(G)$ (b) $H = Z(G)$ (c) H is normal in G (d) H is an abelian group

52. **NET JUNE 2012 (C):** Let G denote the group $S_4 \times S_3$. Then

(a) a 2-Sylow subgroup of G is normal

(b) a 3-Sylow subgroup of G is normal

(c) G has a non-trivial normal subgroup

(d) G has a normal subgroup of order 72

53. **NET DEC 2011 (B):** Let p be a prime number. The order of a p-Sylow subgroup of the group $GL_{50}(\mathbb{F}_p)$ of invertible 50×50 matrices with entries from the finite field $\mathbb{F}_p$, equals:

(a) p^{50} (b) p^{125} (c) p^{1250} (d) p^{1225}

54. **NET DEC 2011 (B):** The number of group homomorphisms from the symmetric group S_3 to $\mathbb{Z}/6\mathbb{Z}$ is

(a) 1 (b) 2 (c) 3 (d) 6

55. **NET DEC 2011 (C):** Which of the following numbers can be orders of permutations σ of 11 symbols such that σ does not fix any symbol?

(a) 18 (b) 30 (c) 15 (d) 28

56. **NET JUNE 2011 (C):** Let $G = \mathbb{Z}_{10} \times \mathbb{Z}_{15}$. Then

(a) G contains exactly one element of order 2

(b) G contains exactly 5 elements of order 3

(c) G contains exactly 24 element of order 5

(d) G contains exactly 24 elements of order 10

57. **NET JUNE 2011 (C):** Let $H = \{e, (1,2)(3,4)\}$ and $K = \{e, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$ be subgroups of S_4, where e denotes the identity element of S_4. Then

(a) H and K are normal subgroups of S_4

(b) H is normal in K and K is normal in A_4

(c) H is normal in A_4 but not normal in S_4

(d) K is normal in S_4, but H is not

58. **NET JUNE 2011 (B):** Let G be a group of order 77. Then center of G is isomorphic to

(a) $\mathbb{Z}$ (b) $\mathbb{Z}_7$ (c) $\mathbb{Z}_{11}$ (d) $\mathbb{Z}_{77}$

59. **NET JUNE 2011 (B):** Consider a group G. Let $Z(G)$ be its center, i.e. $Z(G) = \{g \in G \mid gh = hg \text{ for all } h \in G\}$. For $n \in \mathbb{N}$, the set of positive integers, define

$$J_n = \{(g_1, \dots, g_n) \in Z(G) \times \cdots \times Z(G) \mid g_1 \cdots g_n = e\}.$$

As a subset of the direct product group $G \times \cdots \times G$ (n times direct product of the group G), J_n is

(a) Not necessarily a subgroup

(b) A subgroup but not necessarily a normal subgroup

(c) A normal subgroup

(d) Isomorphic to the direct product $Z(G) \times Z(G) \times \cdots \times Z(G)$ ($(n-1)$ times).

2 Problems on Number Theory (by Dr. J. G. Mehta)

1. **NET JUNE 2019 (C):** Let $a \in \mathbb{Z}$ be such that $a = b^2 + c^2$, where $b, c \in \mathbb{Z} \setminus \{0\}$. Then a cannot be written as

(a) pd^2, where $d \in \mathbb{Z}$ and p is a prime with $p \equiv 1 \pmod 4$

(b) pd^2, where $d \in \mathbb{Z}$ and p is a prime with $p \equiv 3 \pmod 4$

(c) pqd^2, where $d \in \mathbb{Z}$ and p, q are primes with $p \equiv 1 \pmod 4$, $q \equiv 3 \pmod 4$

(d) pqd^2, where $d \in \mathbb{Z}$ and p, q are primes with $p, q \equiv 3 \pmod 4$

2. **NET DEC 2018 (B):** Given integers a and b, let $N_{a,b}$ denote the number of positive integers $k < 100$ such that $k \equiv a \pmod 9$ and $k \equiv b \pmod{11}$. Then which of the following statements is correct?

   (a) $N_{a,b} = 1$ for all integers a and b.

   (b) There exists integers a and b satisfying $N_{a,b} > 1$.

   (c) There exists integers a and b satisfying $N_{a,b} = 0$.

   (d) There exists integers a and b satisfying $N_{a,b} =$ and there exists integers c and d satisfying $N_{c,d} > 1$.

3. **NET DEC 2017 (B):** Let $f : (\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/6\mathbb{Z})$ be the function $f(n) = (n \mod 4, n \mod 6)$. Then

   (a) $(0 \mod 4, 3 \mod 6)$ is in the image of f

   (b) $(a \mod 4, b \mod 6)$ is in the image of f for all even integers a and b

   (c) image of f has exactly 6 elements

   (d) kernel of $f = 24\mathbb{Z}$

4. **NET JUNE 2017 (A):** What is the remainder when 3^{256} is divided by 5?

   (a) 1. (b) 2. (c) 3. (d) 4.

5. **NET JUNE 2017 (B):** Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor by 5. The number of elements in S is

   (a) 480. (b) 420. (c) 360. (d) 240.

6. **NET JUNE 2017 (B):** The remainder obtained when 16^{2016} is divided by 9 equals

   (a) 1. (b) 2. (c) 3. (d) 7.

7. **NET DEC 2016 (B):** Given a natural number $n > 1$ such that $(n-1)! \equiv -1 \pmod n$, we can conclude that

   (a) $n = p^k$ where p is prime, $k > 1$.

   (b) $n = pq$ where p and q are distinct primes.

   (c) $n = pqr$ where p, q, r are distinct primes.

   (d) $n = p$ where p is a prime.

8. **NET JUNE 2016 (B):** Which of the following statements is FALSE? There exists an integer x such that

   (a) $x \equiv 23 \pmod{1000}$ and $x \equiv 45 \pmod{6789}$.

   (b) $x \equiv 23 \pmod{1000}$ and $x \equiv 54 \pmod{6789}$.

   (c) $x \equiv 32 \pmod{1000}$ and $x \equiv 54 \pmod{9876}$.

   (d) $x \equiv 32 \pmod{1000}$ and $x \equiv 44 \pmod{9876}$.

9. **NET DEC 2015 (C):** Which of the following intervals contains an integer satisfying the following three congruences:
$x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$ and $x \equiv 4 \pmod{11}$.
   (a) $[401, 600]$. (b) $[601, 800]$. (c) $[801, 1000]$. (d) $[1001, 1200]$.

10. **NET JUNE 2015 (C):** Which of the following primes satisfy the congruence
$a^{24} \equiv 6a + 2 \mod 13$?
   (a) 41. (b) 47. (c) 67. (d) 83.

11. **NET JUNE 2014 (B):** If n is a positive integer such that sum of all positive integers a satisfying $1 \leq a \leq n$ and $\text{GCD}(a, n) = 1$ is equal to $240n$, then the number of summands, namely, $\phi(n)$, is
   (a) 120. (b) 124. (c) 240. (d) 480.

12. **NET JUNE 2014 (C):** For positive integers m and n, let $F_n = 2^{2^n} + 1$ and $G_m = 2^{2^m} - 1$. Which of the following are true?
   (a) F_n divides G_m whenever $m > n$.
   (b) $\gcd(F_n, G_m) = 1$ whenever $m \neq n$.
   (c) $\gcd(F_n, F_m) = 1$ whenever $m \neq n$.
   (d) G_m divides F_n whenever $m < n$

13. **NET DEC 2013 (B):** For any integers a, b, let $N_{a,b}$ denote the number of positive integers $x < 1000$ such that $x \equiv a \pmod{27}$ and $x \equiv b \pmod{37}$. Then,
   (a) There exists a, b such that $N_{a,b} = 0$.
   (b) For all a, b, $N_{a,b} = 1$.
   (c) For all a, b, $N_{a,b} > 1$.
   (d) There exists a, b such that $N_{a,b} = 1$ and there exists a, b such that $N_{a,b} = 2$.

14. **NET JUNE 2013 (A):** What is the last digit of 7^{73}?
   (a) 7. (b) 9. (c) 3. (d) 1.

15. **NET JUNE 2013 (C):** Consider the congruence $x^n \equiv 2 \pmod{13}$. This congruence has a solution for x if
   (a) $n = 5$. (b) $n = 6$. (c) $n = 7$. (d) $n = 8$.

16. **NET DEC 2012 (B):** The last two digits of 7^{81} are
   (a) 07. (b) 17. (c) 37. (d) 47.

17. **NET DEC 2012 (C):** For positive integers m, let $\phi(m)$ denote the number of integer k such that $1 \leq k \leq n$ and $\text{GCD}(k, m) = 1$. Then which of the following statements are necessarily true?
   (a) $\phi(n)$ divides n for every positive integer n.
   (b) n divides $\phi\big(a^n - 1\big)$ for all positive integers a and n.
   (c) n divides $\phi\big(a^n - 1\big)$ for all positive integers a and n such that $\text{GCD}(a, n) = 1$.
   (d) a divides $\phi\big(a^n - 1\big)$ for all positive integers a and n such that $\text{GCD}(a, n) = 1$.

18. **NET JUNE 2012 (B):** The last digit of $(38)^{2011}$ is

(a) 6. (b) 2. (c) 4. (d) 8.

19. **NET JUNE 2012 (B):** The number of positive divisors of 50000 is

(a) 20. (b) 30. (c) 40. (d) 50.

20. **NET JUNE 2011 (B):** The number of elements in the set $\{m \mid 1 \leq m \leq 1000, m \text{ and } 1000 \text{ are relatively prime}\}$ is

(a) 100. (b) 250. (c) 300. (d) 400.

21. **NET JUNE 2011 (B):** The unit digit of 2^{100} is

(a) 2. (b) 4. (c) 6. (d) 8.

22. **NBHM MSc 2018:** What is the highest power of 3 dividing 1000!?